Perturbation Effects on Mode Propagation in Highly Elliptical Core Two-Mode Fibers

SHANG-YUAN HUANG, JAMES N. BLAKE, AND BYOUNG YOON KIM, MEMBER, IEEE

Abstract—The propagation of both fundamental and second-order modes in highly elliptical core two-mode fibers under external perturbations is investigated. Theoretical and experimental results are presented for the differential phase shift between two spatial modes in each eigenpolarization, and the polarization behavior of each spatial mode. The perturbations include axial strain, radial strain, hydrostatic strain, temperature change, twisting, squeezing, and bending. An application of the modal behavior of these type of fibers to simultaneous measurement of strain and temperature is described.

I. INTRODUCTION

In a two-mode optical fiber with a highly elliptical core, only two orthogonal linearly polarized eigenmodes for each of the fundamental (LP_{01}) and the second-order (LP_{11}) spatial modes can propagate within a large region of the optical spectrum [1], [2]. The two LP_{11} modes have the same lobe orientation so that a stable mode intensity pattern is obtained. With this stability, the performance of recently developed two-mode fiber-optic components, such as acoustooptic frequency shifters [3], intermodal couplers [4], and mode filters [5], will be enhanced greatly, and many new applications, such as interferometric sensors using the two modes as their interferometer arms, can be realized.

The modal behavior of these types of fibers under various external perturbations is of special interest for interferometric sensors and device applications. Most of the work on the wave propagation in elliptical core fibers under external perturbations has been concentrated on the two fundamental polarization modes within the single-mode operating region [6]-[8]. In this paper, we present theoretical and experimental analyses of perturbation effects on all four modes propagating in elliptical core two-mode fibers. The analyses deal with differential phase shifts between the two polarizations of each spatial mode, and between the two spatial modes of each polarization.

We start in Section II by describing two modal birefringences and two polarization birefringences in an elliptical core two-mode fiber. The changes in birefringences and the differential phase shifts between two spatial modes or two polarization modes, induced by radially-symmetric perturbations such as axial strain, radial strain, hydrostatic strain, and the temperature change are formulated. The derivation is based on the assumption of homogeneous elastic and thermal characteristics throughout the fiber. Then, for a twisted elliptical core two-mode fiber, the change of the output pattern with twist rate is discussed. In Section III, the measured modal birefringences and polarization birefringences in unperturbed fibers are presented as functions of optical wavelengths. Then we describe the measured perturbation effects, including axial strain, temperature change, twisting, bending, and squeezing. For the latter two perturbations, no theoretical calculations are presented. These radially asymmetric perturbations make the theoretical consideration much more complicated than symmetric cases. Comparisons of the experimental results with the theoretical calculations show some discrepancies for axial strain and temperature effects. These discrepancies imply a nonuniformity in material characteristics over the fiber cross section and the existence of residual internal stresses in fibers. As for the twist effect, a very low sensitivity of the fiber output pattern to twisting has been found by both theoretical calculations and experimental results. Finally, in Section IV a fiber-optic strain gauge is designed and demonstrated. It is based on the differential phase shift between the two spatial modes induced by an axial strain. This strain gauge provides simultaneous measurements of strain and temperature by using the two polarizations as two independent interferometers.

II. THEORY

A. Modes in Highly Elliptical Core Two-Mode Fiber

A highly elliptical core two-mode fiber guides four nondegenerate eigenmodes with propagation constants, \( \beta_{1x} \), \( \beta_{1y} \), \( \beta_{2x} \), and \( \beta_{2y} \) at a given optical wavelength [1], as illustrated in Fig. 1. The subscripts 1 and 2 correspond to LP_{01} and LP_{11} modes, respectively. The subscripts \( x \) and \( y \) correspond to the major and the minor axes of the fiber core, respectively, ( \( \beta_{1x} > \beta_{1y} \)). The main emphasis of this work is on the characterization of the relative change in propagation constants of the four modes in response to a perturbation. We define the two polarization birefringences \( \Delta\beta_1 = \beta_{1x} - \beta_{1y} \) and \( \Delta\beta_2 = \beta_{2x} - \beta_{2y} \), and two

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S.-Y. Huang was with the Edward L. Ginzton Laboratory, W. W. Hansen Laboratories of Physics, Stanford University, Stanford, CA 94305, on leave from the Department of Electronic Engineering, Shanghai Jiao Tong University, Shanghai, China.

J. N. Blake and B. Y. Kim are with the Edward L. Ginzton Laboratory, W. W. Hansen Laboratories of Physics, Stanford University, Stanford, CA 94305.

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modal birefringences $\Delta \beta _x = \beta _{1x} - \beta _{2x}$ and $\Delta \beta _y = \beta _{1y} - \beta _{2y}$, with four beat lengths $L_{B,i} = \frac{2\pi}{\Delta \beta _i}$ ($i = x, y, 1, 2$) corresponding to them. Only three of the four $\Delta \beta _i$’s are independent of each other as is evidenced by the relationship

$$\Delta \beta _1 - \Delta \beta _2 = \Delta \beta _x - \Delta \beta _y.$$  

These birefringences, $\Delta \beta _i$’s, result in differential phases, $\Delta \phi _i = \Delta \beta _i l$, between two polarization modes or spatial modes after the light propagates along the fiber of length $l$.

**B. Radially Symmetric Perturbation Effects**

Under an external perturbation, the propagation constant in every mode may change due to the modification of the fiber dimensions, the core ellipticity, and the refractive indexes of the core and cladding. Different propagation constant shifts in different modes introduce birefringence changes $\delta (\Delta \beta _i)$.

Then the birefringence change of $\delta (\Delta \beta _i)$ induced by a perturbation $\delta \xi$ is represented by

$$\delta (\Delta \beta _i) = \frac{\partial (n_1 - n_2)}{\partial V} \Delta \beta _i (V) k_0 \delta \xi + \frac{\partial (\Delta \beta _i)}{\partial V} \frac{\partial V}{\partial \xi} \delta \xi.$$  

Using (5) and the relation

$$\frac{\partial (\Delta \beta _i)}{\partial \xi} = \frac{\partial (n_1 - n_2)}{\partial \xi} \frac{V}{\partial \xi} \Delta \beta _i (V) k_0, \quad i = x, y, 1, 2,$$

where $l$ is the fiber length under the perturbation. When $\Delta \phi _i$ and/or $\Delta \phi _j$ change, the intensity distribution of the light exiting the perturbed fiber changes; whereas when $\Delta \phi _1$ and/or $\Delta \phi _2$ change, the output polarization state changes. Utilizing the sensitivities of the $\Delta \beta _i$’s to various perturbations, some special sensors can be designed. In fact, many polarimetric sensors using single-mode fibers have been made by making use of the sensitivity of $\Delta \phi _i$ to environmental perturbations. Considering that modal birefringence is much larger than polarization birefringence, we might expect to get a much higher sensitivity to a perturbation in fiber-optic sensors using $\delta (\Delta \beta _i)$ or $\delta (\Delta \beta _j)$ as opposed to using $\delta (\Delta \phi _i)$ or $\delta (\Delta \phi _j)$.

To calculate the $\delta (\Delta \phi _i)$, we could go through the basic calculation of propagation constants of all modes in the elliptical core two-mode fiber. Unfortunately, a very complicated numerical calculation has to be performed even for an unperturbed elliptical core fiber [9], let alone the task of finding out the differential change of all propagation constants with a perturbation. Considering that all birefringences $\Delta \beta _i$’s in an unperturbed highly elliptical core two-mode fiber could be easily measured, [4], [10], [11] very useful and practical analytic expressions were developed for $\delta (\Delta \beta _i)$ and $\delta (\Delta \phi _i)$ in terms of only $\Delta \beta _i$ and its wavelength dispersion.

For simplicity, we limit our discussion to weakly guiding fibers. Thus we can write the normalized propagation constant $\tilde{\beta} (V)$ as a linear function of propagation constant $\beta$ [12]:

$$\tilde{\beta} (V) = \left( \frac{\beta}{k_0} \right) - \frac{n_2}{n_1 - n_2}$$  

where $k_0$ is the free-space propagation constant, and $n_1$ and $n_2$ are refractive indexes in the core and cladding, respectively. The normalized frequency $V$ has its conventional definition:

$$V = \frac{2\pi}{\lambda} b \sqrt{n_1^2 - n_2^2}$$  

Then the birefringence change of $\delta (\Delta \beta _i)$ induced by a perturbation $\delta \xi$ is represented by

$$\delta (\Delta \beta _i) = \frac{\partial (n_1 - n_2)}{\partial \xi} \Delta \beta _i (V) k_0 \delta \xi + \frac{\partial (\Delta \beta _i)}{\partial V} \frac{\partial V}{\partial \xi} \delta \xi.$$  

Using (5) and the relation

$$\frac{\partial (\Delta \beta _i)}{\partial \xi} = \frac{\partial (n_1 - n_2)}{\partial \xi} \frac{V}{\partial \xi} \Delta \beta _i (V) k_0, \quad i = x, y, 1, 2,$$

where

$$\frac{\partial \beta}{\partial \xi} = \frac{1}{b} \frac{\partial b}{\partial \xi} + \frac{1}{n_1^2 - n_2^2} \left( n_1 \frac{\partial n_1}{\partial \xi} - n_2 \frac{\partial n_2}{\partial \xi} \right).$$  

Combining this result together with expression (2) yields

$$\delta (\Delta \beta _i) = \left[ \frac{1}{n_1 - n_2} \frac{\partial (n_1 - n_2)}{\partial \xi} - \frac{1}{V} \frac{\partial V}{\partial \xi} \right] \Delta \beta _i (V) \frac{l}{\partial \xi} + \frac{\partial (\Delta \beta _i)}{\partial V} \frac{\partial V}{\partial \xi} \delta \xi,$$

$$\delta (\Delta \beta _i) = \left[ -\lambda \frac{\partial \beta}{\partial \xi} \frac{\partial (\Delta \beta _i)}{\partial \xi} \right] \frac{\partial \beta}{\partial \xi} l, \quad i = x, y, 1, 2.$$  

Equations (8) and (10) allow one to calculate the perturbation effect on all $\Delta \beta _i$’s and all the differential phase shifts between the various modes using experimentally measured values of the unperturbed $\Delta \beta _i$’s and their wavelength dispersions. Note that the use of (7) in the calculation of perturbation effects implies that the dispersion curves of normalized propagation constants in all modes remain unchanged in spite of the perturbation, while only
the operating point will be adjusted with the perturbations due to the changes of the fiber dimension and refractive indexes in the core and cladding. In fact, the normalized dispersion curves of a weakly guiding fiber depend only on the core ellipticity provided that no internal stress exists in the fiber [13]. Therefore, (8)-(10) hold for cases in which the core ellipticity remains constant under the perturbation. In other words, two conditions are required to use (8)-(10). First, the perturbation should be isotropic over the fiber cross section. Second, the fiber has homogeneous elastic and thermal characteristics so that no anisotropic transverse strains and refractive index changes will be induced. For anisotropic perturbations such as squeezing and bending, the ellipticity of the fiber core changes with perturbation strength. In this case, the measured dispersion curve of the fiber under the original ellipticity does not provide enough information for predicting the perturbation effects.

The effects of some isotropic perturbations on a weakly guiding two-mode fiber are calculated as follows. They include axial strain, radial strain, hydrostatic strain, and temperature changes. It can be seen from (8)-(10) that only \( \delta b / \delta \xi, \delta a / \delta \xi, \text{ and } \delta n_j / \delta \xi \) need to be found to obtain \( \delta (\Delta \beta_i) / \delta \xi \) and \( \delta (\Delta \phi_j) / \delta \xi \).

1) Axial Strain: Assume that the fiber is a homogeneous material in elasticity, i.e., the Young’s modulus \( E \) and Poisson’s ratio \( \nu \) are considered to be the same for both the core and cladding. The fiber is stretched by an amount \( \delta l \) over a fiber length \( l \) (\( \delta l \ll l \)). In this case, \( \delta \xi = \delta l \). For the semiminor axis of the core, \( b \), and the refractive indexes in the core and cladding, \( n_j \), we have

\[
\frac{\partial b}{\partial l} = -\nu \frac{b}{l} \tag{11}
\]

and

\[
\frac{\partial n_j}{\partial l} = -\frac{1}{2l} n_j \left[ p_{12} - (p_{11} + p_{12}) \nu \right], \quad j = 1, 2 \tag{12}
\]

where \( \nu \) is the Poisson’s ratio (0.17 for fused silica), and \( p_{11} \) and \( p_{12} \) are the strain optic coefficients (0.12 and 0.27, respectively, for fused silica, at \( \lambda = 633 \text{ nm} \)). Inserting (11) and (12) into (8) and (9), and taking the above values yields

\[
\delta (\Delta \beta_i) = 0.170 - 0.102\bar{n}^2 \frac{\delta \beta_i}{l} + (0.17 + 0.204\bar{n}^2) \lambda \frac{\delta (\Delta \beta_i)}{\delta \lambda} \frac{\delta l}{l} \tag{13}
\]

where \( \bar{n} \) is the average index of the core and cladding. Taking \( \bar{n} = 1.46 \), (13) becomes

\[
\frac{\delta (\Delta \beta_i)}{\delta l} = -0.047\Delta \beta_i + 0.605\lambda \frac{\delta (\Delta \beta_i)}{\delta \lambda} \tag{14}
\]

Similarly, we have

\[
\frac{\delta (\Delta \phi_j)}{\delta l} = 0.953\Delta \beta_i + 0.605\lambda \frac{\delta (\Delta \beta_i)}{\delta \lambda} \tag{15}
\]

2) Radial Strain: Assume that a fiber experiences a uniform, compressional, radial pressure \( \delta P \), and the axial stress over the ends of the fiber is zero. For this case, \( \delta \xi = \delta P \). The radial and longitudinal strains are given by

\[
\frac{\partial b}{b} = -(1 - \nu) \frac{\delta P}{E} \quad \text{and} \quad \frac{\partial l}{l} = 2\nu \frac{\delta P}{E}. \tag{16}
\]

The differential refractive indexes are given by

\[
\frac{\partial n_j}{\partial P} = \frac{n_j^3}{2E} \left[ \left( p_{11} + p_{12} \right) (1 - \nu) - 2p_{12} \right], \quad j = 1, 2. \tag{17}
\]

Using (8)-(10) and taking the above-mentioned constants yields

\[
\frac{\delta (\Delta \beta_i)}{\delta P} = \left[ 1.077\Delta \beta_i + 0.336\lambda \frac{\delta (\Delta \beta_i)}{\delta \lambda} \right] \frac{1}{E} \tag{18}
\]

and

\[
\frac{\delta (\Delta \phi_j)}{\delta P} = \left[ 1.417\Delta \beta_i + 0.336\lambda \frac{\delta (\Delta \beta_i)}{\delta \lambda} \right] \frac{1}{E} \tag{19}
\]

where the Young’s modulus \( E \) is taken to be \( 7 \cdot 10^{10} \text{ N/m}^2 \).

3) Hydrostatic Strain: Consider a fiber under isotropic stress due to a compressional pressure \( \delta P \), and the axial stress over the fiber ends is \( -\delta P \). We have \( \delta \xi = \delta P \). A uniform strain will be obtained throughout the fiber:

\[
\frac{\partial b}{b} = \frac{\partial l}{l} = -(1 - \nu) \frac{\delta P}{E}. \tag{20}
\]

Also, we have

\[
\frac{\partial n_j}{\partial P} = \frac{n_j^3}{2E} \left( 1 - 2\nu \right) \left( p_{11} + 2p_{12} \right), \quad j = 1, 2. \tag{21}
\]

Finally, we obtain

\[
\frac{\delta (\Delta \beta_i)}{\delta P} = \left[ 0.366\Delta \beta_i - 0.099\lambda \frac{\delta (\Delta \beta_i)}{\delta \lambda} \right] \frac{1}{E} \tag{22}
\]

and

\[
\frac{\delta (\Delta \phi_j)}{\delta P} = \left[ -0.464\Delta \beta_i - 0.099\lambda \frac{\delta (\Delta \beta_i)}{\delta \lambda} \right] \frac{1}{E}. \tag{23}
\]

4) Temperature Change: A change in temperature causes a change in the dimensions of the fiber and in the refractive indexes. Under the assumption of homogeneous thermal characteristics, the core and cladding have the same thermal expansion coefficient \( \alpha \) and the same thermooptic coefficient \( \xi \). Take the \( \delta \xi \) as the temperature change \( \delta T \) whose sign is considered to be positive upon heating. We have

\[
\frac{\partial b}{\partial T} = \alpha b \quad \text{and} \quad \frac{\partial l}{\partial T} = \alpha l \tag{24}
\]

as well as

\[
\frac{\partial n_j}{\partial T} = \xi n_j, \quad j = 1, 2. \tag{25}
\]
Using (8)-(10) yields
\[
\frac{\delta(\Delta \beta_i)}{\delta T} = -\alpha \Delta \beta_i - \lambda \frac{\partial (\Delta \beta_i)}{\partial \lambda} (\alpha + \gamma),
\]
\[i = x, y, 1, 2 \tag{26}
\]
and
\[
\frac{\delta(\Delta \phi_i)}{\delta T} = -\lambda \frac{\partial (\Delta \beta_i)}{\partial \lambda} l(\alpha + \gamma),
\]
\[i = x, y, 1, 2 \tag{27}
\]
For fused silica, \(a \approx 5 \times 10^{-7}/\text{C}\) and \(\xi \approx 1 \times 10^{-5}/\text{C}\). Equation (27) shows that \(\delta(\Delta \phi_i)/\delta T\) depends upon only the wavelength dispersion of \(\Delta \beta_i\). An important point to notice is that the temperature sensitivity is zero when \(\partial (\Delta \beta_i) / \partial \lambda = 0\), where the LP_{01} and LP_{11} modes have the same group velocity \([4]\).

C. Twisting Effect

A straight fiber with a highly elliptical core supports linear polarization eigenstates, whose polarization directions coincide with the principal axes of the ellipse. Consequently, a linearly polarized input along either axis maintains its polarization state along the length of the fiber. When the fiber is twisted, however, the output polarization state is changed due to both a geometrical effect and an elastooptic effect. The polarization evolution of the fundamental mode in a twisted single-mode fiber was obtained by using either the Poincare’ sphere \([8]\) or the coupled-mode equations \([6], [14]\).

Let us consider a twisted highly elliptical core two-mode fiber. We define the local coordinates \(x', y'\) which follow the principal axes of the twisted fiber. Assume that there is no coupling between the fundamental and second-order modes caused by the twisting process such that we could treat the fundamental mode and the second-order mode independently. This assumption is valid since pure twisting does not break the symmetry of the fiber core with respect to its major and minor axes. Considering the polarization evolution theory used for single-mode fibers, we can obtain the sensitivity of \(l\) to the change of \(\phi\), which describes the twisting effect on the two-mode interference pattern. This sensitivity is dependent upon the fiber length \(l\) and all birefringences \(\Delta \beta_i\) (\(i = x, y, 1, 2\)).

Finally, let us consider the photoelastic coefficients \(g_i\) and \(g_2\) for a fiber with a circular core for a simplicity of calculation. In a weakly-guiding twisted fiber, the perturbed transverse electric fields can be represented as \([8]\)
\[
E^{(i)}_r(x, y, z) = \sum_{m=x,y} a^{(i)}_m(z) \exp(jk_z^{(i)}z) E^{(i)}_m(x, y),
\]
\[i = 1, 2. \tag{32}
\]

The symbol \(i\) denotes the mode order. \(E_r(x, y)\) and \(E'_r(x, y)\) are the transverse field components corresponding to a pair of orthogonal unperturbed LP modes. \(k_1\) and \(k_2\) are the propagation constants of these pairs of orthogonal LP modes. Under weakly-guiding unperturbed conditions, we have \(k_1^{(i)} = k_2^{(i)}\). The field amplitudes \(a_m(z)\) satisfy the coupled-mode equations
\[
\frac{d a_m^{(i)}(z)}{dz} = j \sum_{n=1,2} k_m^{(i)} a_n^{(i)}(z),
\]
\[i = 1, 2 \quad (m = x, y) \tag{33}
\]
with
\[
K_i = \sqrt{(\Delta \beta_i)^2 + (\phi(2 - g_i))^2}, \quad i = 1, 2. \tag{30}
\]
Here \(l\) is the fiber length, and \(g_i\) is a photoelastic coefficient for the \(i\)th mode. The term \(g_2\) is often called optical activity. The \(E_{10}^{(i)}\) and \(E_{20}^{(i)}\) are the electric fields in the two polarizations for the \(i\)th mode, at the input end of the fiber. Since the second-order mode has antisymmetric field (see Fig. 1), we can take either half of the fiber cross section as a phase-reference at the input. Then look at the half cross section all the way from this reference half-section, following the twisted local coordinate system. In this way, no phase distribution needs to be taken into account.

Suppose both spatial modes are excited at the input end of the fiber. As mentioned above, we look at only half of the fiber cross section, following the local coordinate system along the fiber. The intensity in this half of the fiber cross section at the output end \(l\) is expressed as
\[
l = I_r + I_r = |E_r^{(1)}(1) + E_r^{(2)}(2)|^2 + |E_r^{(1)}(1) + E_r^{(2)}(1)|^2 \tag{31}
\]
where \(I_r\) and \(I_r\) are the intensity components in \(x\) and \(y\) coordinates, and \(E_r^{(1)}(1)\) and \(E_r^{(1)}(1)\) are represented in (28) and (29). It can be seen that \(l\) is a function of the twist rate \(\phi\). By using (31), we can obtain the sensitivity of \(l\) to the change of \(\phi\), which describes the twisting effect on the two-mode interference pattern. This sensitivity is dependent upon the fiber length \(l\) and all birefringences \(\Delta \beta_i\) (\(i = x, y, 1, 2\)).

The calculation results show that both

\[
E_r^{(i)}(x, y) = e^{-j\theta_0}(2b + b_0) \left[ E_{10}^{(i)} \cos \frac{K_1 l}{2} - \frac{j \Delta \beta_i}{K_1} E_{20}^{(i)} \sin \frac{K_1 l}{2}
\]
\[
+ \frac{\phi(2 - g_i)}{K_1} E_{20}^{(i)} \sin \frac{K_1 l}{2} \right] \tag{28}
\]
and
\[
E_r^{(i)}(x, y) = e^{-j\theta_0}(2b + b_0) \left[ E_{10}^{(i)} \cos \frac{K_1 l}{2} + \frac{j \Delta \beta_i}{K_1} E_{20}^{(i)} \sin \frac{K_1 l}{2}
\]
\[
- \frac{\phi(2 - g_i)}{K_1} E_{20}^{(i)} \sin \frac{K_1 l}{2} \right] \tag{29}
\]
perturbation effects on highly elliptical core two-mode fibers

1. INTRODUCTION

Highly elliptical core two-mode fibers have been shown to have a large modal birefringence and a small second mode cutoff, which make them suitable for optical communications systems, where rapid and accurate fiber sorting is critical. In this paper, we report on the effects of perturbations on the modal birefringence of highly elliptical core two-mode fibers. We show that the modal birefringence can be significantly increased by applying a twist to the fiber, and that this effect can be used to sort fibers into different polarization states.

2. METHODS

Our measurements were carried out on two fibers with different core dimensions: Fiber A had a core ellipse of 4.1 μm x 2.2 μm (provided by Polaroid Corporation) and Fiber B had a core ellipse of 2 μm x 1 μm (provided by Andrew Corporation). Both fibers had a second mode cutoff λ ~ 633 nm. The modal beat lengths were measured using the method described in [4]. The spatial distributions of the light were measured using a coherent superposition of the light contained in the symmetric LP_{01} mode and the antisymmetric LP_{11} mode, being a function of the relative polarization beat lengths.

3. RESULTS

Figure 2 shows the measured dependence of Δβ_{1} and Δβ_{2}, on optical wavelength λ for Fiber A and Fiber B. As can be seen from the curves, both Δβ_{1} and Δβ_{2} decrease with increasing optical wavelength. With these polarization measurements, the small differences between Δβ_{1} and Δβ_{2} (i.e., the difference between Δβ_{2} and Δβ_{1}) can now be seen for the two fibers. The LP_{01} mode shows a slightly smaller polarization birefringence than the LP_{11} mode. The difference between Δβ_{1} and Δβ_{2} increases with increasing λ.

4. DISCUSSION

The polarization beat lengths, hence, the polarization birefringences, in the two spatial modes can be measured by the conventional beat pattern measurement with scattered light. A linearly polarized LP_{01} mode (or LP_{11} mode) is excited with a 45° polarization direction with respect to the fiber eigenaxes. Looking at the fiber in the direction normal to the propagation axis, a series of dark and bright bands appear in the scattered light, with a period of L_{B,1} (or L_{B,2}). The experimental curves of Δβ_{1} and Δβ_{2} versus λ for both fibers are shown in Fig. 3. As can be seen from the curves, both Δβ_{1} and Δβ_{2} decrease with increasing optical wavelength. With these polarization measurements, the small differences between Δβ_{1} and Δβ_{2} (i.e., the difference between Δβ_{2} and Δβ_{1}) can now be seen for the two fibers. The LP_{11} mode shows a slightly smaller polarization birefringence than the LP_{01} mode. The difference between Δβ_{1} and Δβ_{2} increases with increasing λ.

5. CONCLUSION

In conclusion, we have shown that the modal birefringence of highly elliptical core two-mode fibers can be significantly increased by applying a twist to the fiber. This effect can be used to sort fibers into different polarization states, which is important for optical communications systems. Further research is needed to understand the mechanism behind this effect, and to develop practical methods for sorting fibers using this effect.
phase difference between the two modes $\Delta \Phi_x$ or $\Delta \Phi_y$. Therefore, observing the output radiation pattern on the screen, one can determine the sensitivity of $\Delta \Phi_x$ and $\Delta \Phi_y$ to the perturbation.

1) Axial Strain: For this test, the perturbed fiber section of length $l$ in Fig. 4 was stretched. With increasing fiber elongation $\delta l$, a periodic pattern oscillation from side to side along the major axis of the core was observed on the screen, for both Polaroid and Andrew fibers [16]. $\delta (\Delta \Phi_x)/\delta l$ and $\delta (\Delta \Phi_y)/\delta l$ were calculated from the measurements of $2\pi/\delta l_{x,2}$ and $2\pi/\delta l_{y,2}$, respectively. Here $\delta l_{x,2}$ and $\delta l_{y,2}$ were the fiber elongations needed to produce a complete oscillation on the screen. Fig. 5 presents the experimental values of $\delta (\Delta \Phi_x)/\delta l$ and $\delta (\Delta \Phi_y)/\delta l$ as functions of $\lambda$ for the Polaroid fiber. The $y$-polarization exhibits a higher sensitivity than the $x$-polarization. A theoretical curve, calculated by using (15) and the data of the Polaroid fiber shown in Fig. 2, is also plotted in Fig. 5. Since the measured $\Delta \Phi_x$ and $\Delta \Phi_y$ are essentially equal, only one theoretical curve is shown for both polarizations. The theoretical curve has a wavelength dependence similar to the experimental values and is closer in magnitude to the measured value of the $y$-polarization.

In order to better understand the polarization splitting observed in the experiment, we investigated the polarization behavior of both the Polaroid and Andrew fibers under axial strain. By observing the scattering beat pattern, we measured the polarization birefringences $\Delta \beta_1$ and $\Delta \beta_2$ for the fibers under stretching. Fig. 6 shows $\Delta \beta_1$ and $\Delta \beta_2$ as functions of the axial strain $\delta l/l$, at $\lambda = 514.5$ nm, for the Polaroid and Andrew fibers, respectively. Here $l$ is the fiber length under stretching and $\delta l$ is the fiber extension. The curves show a linear dependence of $\Delta \beta_1$ and $\Delta \beta_2$ on $\delta l/l$. However, their slopes have opposite signs. For the Polaroid fiber, we have $\delta (\Delta \beta_1)/\delta l/l = -0.39$ rad/mm and $\delta (\Delta \beta_2)/\delta l/l = 6.24$ rad/mm at $\lambda = 514.5$ nm. For the Andrew fiber, we have $\delta (\Delta \beta_1)/\delta l/l = -13.16$ rad/mm and $\delta (\Delta \beta_2)/\delta l/l = 17.84$ rad/mm at $\lambda = 514.5$ nm. Both fibers under test show that an axial strain reduces the polarization birefringence of the LP$_{01}$ mode, but increases the polarization birefringence of the LP$_{11}$ mode.

Inserting the measured $\Delta \beta_1$, $\Delta \beta_2$ and their wavelength dispersion values of unperturbed Polaroid fibers into (14), we can get the theoretical prediction for $\delta (\Delta \beta_1)/\delta l/l$ and $\delta (\Delta \beta_2)/\delta l/l$. Table I gives the calculated and measured values for the Polaroid fiber at two optical wavelengths. Table I shows that measured $\delta (\Delta \beta)/\delta l/l$'s are quite different from the calculated values. We think this indicates an inhomogeneity in the elastic characteristics of those fibers. An anisotropic transverse strain distribution over the cross section of a fiber is introduced when the fiber is strained longitudinally. The difference in Young's modulus and Poisson's ratios of the core and cladding, that produces transverse stress when the fiber is longitudinally

![Fig. 4. Scheme of the experimental arrangement to measure perturbation effects on the modal differential phase.](image-url)

![Fig. 5. Wavelength dependence of $\delta (\Delta \Phi_x)/\delta l$ and $\delta (\Delta \Phi_y)/\delta l$ in Polaroid fibers. Broken line represents the theoretical result based on the measured $\Delta \Phi_x (= \Delta \Phi_y)$ and its wavelength dispersion. Solid lines represent measured results for $x$- and $y$-polarizations.](image-url)

![Fig. 6. Measured $\Delta \beta_1$ and $\Delta \beta_2$ as functions of the axial strain $\delta l/l$ for (a) a Polaroid fiber and (b) an Andrew fiber.](image-url)

![Table I](image-url)
strained, may have to be taken into account for an accurate theoretical prediction. From the expression
\[
\delta = (\Delta \beta_1 - \Delta \beta_2) + \left[ \delta(\Delta \beta_1)/\delta l/1 - \delta(\Delta \beta_2)/\delta l/1 \right] (38)
\]
we obtain the above-mentioned polarization splitting in modal behavior which is due to the big difference between \(\delta(\Delta \beta_1)/\delta l/1\) and \(\delta(\Delta \beta_2)/\delta l/1\).

Different fiber structures and different dopants may provide different \(\delta(\Delta \beta)/\delta l/1\)’s. For elliptical core fibers, a higher dopant concentration (e.g., Andrew fiber) exhibited larger absolute values of \(\delta(\Delta \beta_1)/\delta l/1\) and \(\delta(\Delta \beta_2)/\delta l/1\) because of the larger difference of Poisson’s ratios. Stress-induced birefringent fibers usually provide much higher sensitivity of polarization birefringence to stretching. It has been reported that stretching stress-induced birefringent single-mode fibers showed a \(\delta(\Delta \beta_1)/\delta l/1 \sim 33 \text{ rad/mm} [17]\) for an elliptically-jacketed fiber, a \(-64 \text{ rad/mm} [18]\) for an elliptically-clad fiber, and \(-42 \text{ rad/mm} [17] \sim 120 \text{ rad/mm} [19]\) for bowtie fibers. The operating optical wavelength in [17] and [18] is 633 nm.

A new fiber-optic strain gauge using \(\delta(\Delta \phi_1)/\delta l\) and \(\delta(\Delta \phi_2)/\delta l\) is described in Section IV. The unique polarization splitting in mode behavior can be advantageously utilized to make new types of fiber-optic components such as polarization filters, wavelength filters, and polarization controllers.

2) Temperature Change: Using the setup shown in Fig. 4, the dependence of modal differential phase on temperature changes was measured. The experiments were carried out by slowly heating a Polaroid fiber sample (\(-3.5 \text{ m}\)) and an Andrew fiber sample (\(-0.9 \text{ m}\)) in a water bath. The change of the far-field radiation pattern with temperature changes was observed, yielding values of \(\delta(\Delta \phi)/\delta T\). Measurements were performed for both the \(x\)-polarization and \(y\)-polarization. Fig. 7 shows the experimental curves of \(\delta(\Delta \phi_1)/\delta T\) and \(\delta(\Delta \phi_2)/\delta T\) per meter of Polaroid fiber versus wavelength \(\lambda\). Also included in this figure is the theoretical prediction of \(\delta(\Delta \phi_1)/\delta T\) or \(\delta(\Delta \phi_2)/\delta T\) from (27). Since \(\delta(\Delta \phi_1)/\delta \lambda\) and \(\delta(\Delta \beta)/\delta \lambda\) were measured to pass through zero near \(\lambda = 514.5 \text{ nm}\), the Polaroid fiber was predicted to be temperature independent in modal behavior at this wavelength. We remark on two characteristics of the curves in Fig. 7. The first is that the measured curves of \(\delta(\Delta \phi_1)/\delta T\) and \(\delta(\Delta \phi_2)/\delta T\) are approximately parallel, but with an offset, to the theoretical curve. We believe that the offset between the measured values and the predicted values is caused by a relieving of the residual internal stresses developed in the manufacturing process. Temperature sensors using elliptical core two-mode fibers may be practical using a fiber having a large residual thermal stress. The second is that temperature changes affect the two polarizations slightly differently. The source of this polarization splitting is probably due to the thermal stresses relieving asymmetrically. These points again reveal the nonuniformity of thermal and elastic characteristics over the fiber cross section due to the expansion coefficient mismatch between the fiber core and cladding.

Table II lists some measured values of the characteristic temperature change \(\Delta T_{\phi}\) which produces a 2\(\pi\) differential phase shift between \(L_{S0}\) and \(L_{S1}\) modes, for a slightly doped fiber (Polaroid) and a highly doped fiber (Andrew), respectively. As can be seen from Table II, mode propagations in the highly doped fiber show a much higher sensitivity to temperature. This is due to the much stronger residual thermal stresses present in the highly doped fiber than in the slightly doped fiber. A fiber heating test on polarization behavior in each spatial mode has been also performed. At \(\lambda = 514.5 \text{ nm}\), we measured the change of \(\Delta \beta_1\) and \(\Delta \beta_2\) with temperature for a Polaroid fiber by observing the scattering polarization beat patterns of the \(L_{S0}\) and \(L_{S1}\) modes, respectively. We measured \(d(\Delta \beta_1)/dT \sim -0.16 \text{ rad/m°C}\) and \(d(\Delta \beta_2)/dT \sim -0.18 \text{ rad/m°C}\). The negative signs indicates a relieving of the thermal stresses with heating. This result is consistent with the measurement of modal birefringence shown in Fig. 7.

3) Twist: The twisting effect on polarization behavior and modal behavior in highly elliptical core two-mode fibers was observed using the setup shown in Fig. 8. The 

![Fig. 7. Wavelength dependence of \(\delta(\Delta \phi_1)/\delta T\) and \(\delta(\Delta \phi_2)/\delta T\) for 1 m of Polaroid fiber. Broken line represents the theoretical result based on the measured wavelength dispersion of \(\Delta \beta_1\) and \(\Delta \beta_2\). Solid lines represent measured results for \(x\)- and \(y\)-polarizations.](image-url)
input end of the fiber was bonded to a fixed post, while the output end was bonded to a rotation stage. At first, we observed the polarization evolution in each spatial mode. We first excited only the LP(21) mode in a 19.3-cm-long Polaroid fiber before twisting, at \( \lambda = 514.5 \text{ nm} \). Then the output end of this fiber was twisted gradually. It can be seen from (28), (29) that the intensity component \( I_{y'}^{(1)} = |E_{y'}^{(1)}|^2 = 0 \) and \( I_x^{(1)} = |E_x^{(1)}|^2 = 1 \) will occur at such twist rates \( \phi \)'s that satisfy the relation

\[
\sin \left( \frac{(\Delta \beta_1)^2}{2} \right) \left[ (2 - g_1) \right]^2 = 0. \tag{39}
\]

The intensity \( I_y^{(1)} \) was monitored with a polarization analyzer whose transmission direction followed the local fiber axis \( y' \). A number of \( \phi \)-values making \( I_y^{(1)}(\phi) = 0 \) were measured and compared with the theoretical values from (39). In the twisting process, little mode coupling between LP_{01} and LP_{11} modes took place. Taking \( g_1 = 0.16 \) and the measured value \( \Delta \beta_1 = 0.311 \text{ rad/mm} \), a normalized theoretical curve of \( I_y^{(1)} \) versus twist rate \( \phi \) for the Polaroid fiber sample under test is calculated by using (29), (30) (taking \( E_{x0}^{(1)} = 1 \) and \( E_{y0}^{(1)} = 0 \)), and plotted in Fig. 9. The measured \( \phi \) values are also shown in Fig. 9. An excellent agreement for the nulling points between theory and experiment can be found even with a twist rate exceeding 300 rad/m. The same experiment for the second-order mode was done, and a good agreement between theory and experimental results was obtained also.

Then, we investigated the modal behavior. A linearly-polarized beam was launched into a straight Polaroid fiber with an arbitrary azimuth. We adjusted the launching condition to get an approximately 50-percent excitation in the LP_{01} and LP_{11} modes, respectively. As the fiber was twisted, the far-field radiation pattern was observed on a screen (without a polarization analyzer after the fiber). The experiments were carried out for two different wavelengths (514.5 and 591 nm) of argon laser output. As can be seen in Fig. 3, the polarization birefringences for the two spatial modes (\( \Delta \beta_1 \) and \( \Delta \beta_2 \)), have a greater difference at 591 nm than at 514.5 nm. It is expected that the effects of twist for the two wavelengths are different. For a \( \sim 0.5 \text{-m-long} \) Polaroid fiber at \( \lambda = 514.5 \text{ nm} \), a twist up to 10 turns did not produce a visual change in the output pattern, except that the output pattern rotated with the output endface of the fiber. At \( \lambda = 591 \text{ nm} \), on the other hand, about 17–18 turns of twist were required to make a complete power-transfer from the reference side to the other. This observation can be analyzed by using (31) and (28)–(30). Suppose the two spatial modes with equal intensities are in phase in one half of the fiber cross section at the input end. The symmetric first mode and the anti-

symmetric second mode will interfere constructively on this half and destructively on the other half. In this case, we have \( E_{x0}^{(1)} = E_{x0}^{(2)} = E_x \) and \( E_{y0}^{(1)} = E_{y0}^{(2)} = E_y \) on the constructive side (or reference half), with \( |E_{x0}|^2 + |E_{y0}|^2 = 1/2 \) for normalizing the total intensity. We have calculated the intensity in the reference half-section at the fiber output-end \( I \) as a function of the twist rate, \( \phi \), for Polaroid fibers. In Fig. 10(a), we show two curves, 1 and 2, corresponding to \( \lambda = 514.5 \text{ nm} \) and 591 nm, respectively. The data of \( \Delta \beta_1 (i = x, y, 1, 2) \) used in the calculations were taken from the measured results shown in Figs. 2 and 3. The fiber length was taken around 0.5 m with a fine adjustment to make \( \phi \) approximately zero (i.e., get darkness on the reference half in the fiber output-pattern) in the absence of a twist. These curves were obtained by taking \( E_{x0} = 1/\sqrt{2} \) and \( E_{y0} = 0 \). As can be seen from Fig. 10(a), at \( \lambda = 514.5 \text{ nm} \) (curve 1), only 10 percent of the total power is transferred from one side to the other when the \( \sim 0.5 \text{-m-long} \) fiber is twisted with \( \phi \sim 125 \text{ rad/m} \) (about 10 turns). Even under a twist rate of \( \sim 800 \text{ rad/m} \) for this fiber sample (about 64 turns), only less than 40 percent of the total power can be transferred.

At \( \lambda = 591 \text{ nm} \), a larger difference of \( \Delta \beta_1 \) and \( \Delta \beta_2 \) than that of \( \lambda = 514.5 \text{ nm} \) is obtained (see Fig. 3), leading to a stronger effect. This can be seen from curve 2 in Fig. 10(a). The curve shows that a twist rate of \( \sim 220 \text{ rad/m} \) (about 17.5 turns) is needed to get a full power transfer from one half to the other half in the radiation pattern, which agrees well with the measured value. One important point shown by calculations and experiments is that the sensitivity of \( I \) to \( \phi \) is independent of the ratio between \( E_{x0} \) and \( E_{y0} \). Therefore, the curves in Fig. 10(a) are valid for any combination of \( E_{x0} \) and \( E_{y0} \). Fig. 10(b) shows an expanded view of a portion of curve 1 in Fig. 10(a) with the intensity components in the \( x' \) and \( y' \) coordinates, \( I_{x'} \) and \( I_{y'} \). The \( I_{x'} \) and \( I_{y'} \) show fast oscillations with twisting although their sum is smooth.

Similar behavior was observed for the Andrew fiber. Since the Andrew fiber is highly birefringent, the first and second modes themselves are quite stable against the twist perturbation.

4) Other Perturbations: Squeezing and bending tests were carried out using the setup shown in Fig. 4. To avoid
an output pattern change induced by polarization coupling, the squeezing and bending were controlled along the x- or y-eigendirections. The perturbations were applied near the fiber output end for an easy determination of the eigendirections from the output pattern. Both Polaroid and Andrew fibers were tested. Little output pattern change was found, when a weight of more than 10 kg was pressed upon a section of 10-cm-long fiber, or when a small coil with a diameter about 1 cm was formed. This means that the LP_{01} and LP_{11} modes have almost the same phase shift when the fiber is squeezed or bent.

For a radial or a hydrostatic strain, experimental work has not been carried out yet. However, using the result obtained in Section II, we have made numerical estimations for the Polaroid fiber and compare the sensitivity of the differential phase between the two spatial modes with the sensitivity of an interferometer with two separate fiber paths. First, consider a Mach–Zehnder interferometer formed by single-mode fibers. By a treatment similar to calculating $\delta(\Delta \phi_y)/l$$b_0$ in Section II, we have calculated the pressure sensitivity in a single mode fiber $\delta \phi_1/l$$b_0$. The parameters of the single-mode fiber and the operating condition used for calculation are as follows: core index $n_1 = 1.47$, cladding index $n_2 = 1.457$, optical wavelength $\lambda = 0.633 \mu m$, normalized frequency $V = 2.4$, normalized propagation constant $\bar{b} = 0.5$, and $\delta b/\delta V = 0.3$, together with the all elastooptic parameters as taken in Section II-B. The calculated sensitivities are $\delta \phi_1/l$$b_0 \sim 1.2 \times 10^{-4}$ rad/Pa m and $\delta \phi_1/l$$b_0 \sim -7.5 \times 10^{-5}$ rad/Pa m, for a radial and a hydrostatic pressure, respectively. Then consider the sensitivities in differential phase between two spatial modes in a Polaroid fiber. Inserting the measured numbers at $\lambda = 514.5$ nm, $\Delta \phi_i (i = x, y) \sim 30.8$ rad/mm and $\delta(\Delta \phi_i)/\delta \lambda (i = x, y) \sim 0$, into (19), (23), we have obtained $\delta(\Delta \phi_i)/l$$b_0 (i = x, y) \sim 6 \times 10^{-7}$ rad/Pa m for a radial strain, which is about 200 times smaller than the conventional single-mode fiber Mach–Zehnder scheme; and $\delta(\Delta \phi_i)/l$$b_0 (i = x, y) \sim -2 \times 10^{-7}$ rad/Pa m for a hydrostatic strain, which is about 370 times lower than the Mach–Zehnder scheme.

IV. STRAIN GAUGE USING TWO-MODE FIBERS

By using the axial strain effect on the differential phase shift between the two spatial modes in a highly elliptical core two-mode fiber, an experimental strain gauge has been demonstrated. The configuration is depicted in Fig. 11. Light from an argon laser ($\lambda = 514.5$ nm) is launched approximately equally into both the first- and second-order modes and also into each eigenpolarization. Light launched into one-half of the fiber core polarized at $45^\circ$ from the ellipse axes will accomplish this. The light then passes through the sensing region and the eigenpolarizations are separated at the output, with a simple beamsplitter and polarizers or a polarization beamsplitter. Offset detectors then detect the interfering of the LP_{01} and LP_{11} modes for their respective polarizations. The fiber thus acts as two independent spatial mode interferometers, one for each polarization state.

Under a stable ambient temperature, the light intensity at each detector is given by

$$I' \sim 1 + C \cos \left(2\pi \frac{\delta l}{\delta l_{12}} + \phi_0\right)$$

where $i = x, y$ is the polarization being detected, $\delta l$ is the fiber elongation, $\phi_0$ is the unperturbed phase difference between the two spatial modes at the fiber end, and $C ( < 1)$ is the electrical fringe visibility, dependent on the launching conditions and the detector area and location. Fig. 12 shows plots of $I'$ and $I''$ versus $\delta l$ recorded simultaneously during a Polaroid fiber stretched by a linear translation stage. As can be seen the period of oscillation is different for the two polarizations.

By making use of these two interferometers, it is possible to monitor the strain and the ambient temperature simultaneously. The change in the relative phase between the first- and second-order modes for each polarization, $\delta(\Delta \phi_i)$ and $\delta(\Delta \phi_{12})$, due to both length and temperature changes $\delta l$ and $\delta T$ can be expressed as

$$\begin{bmatrix} \delta(\Delta \phi_i) \\ \delta(\Delta \phi_{12}) \end{bmatrix} = \begin{bmatrix} 2\pi/\delta l_{12} & 2\pi/\delta T_{12} \\ 2\pi/\delta l_{12} & 2\pi/\delta T_{12} \end{bmatrix} \begin{bmatrix} \delta l \\ \delta T \end{bmatrix}.$$
Fig. 11. Experimental strain gauge; P: polarizer, BS: beam splitter, A,, A,-: polarization analyzers, D,, D,-: detectors.

Fig. 12. Intensities, I,, and I,., monitored at D,, and D,-, as a function of fiber extension.

As long as the determinant of the matrix is nonzero, this equation can be inverted and both δl and δT can be explicitly found from the measured values of δ(Δφ,.) and δ(Δφ,). Thus we find that ambient temperature changes and strain can be independently and simultaneously measured because strain and temperature changes affect the two polarization interferometers differently. Mach–Zehnder interferometers [21] and polarimetric strain sensors [18] typically have large sensitivities to strain which unfortunately cannot be differentiated from temperature changes. This limits the practicality of such types of strain gauges. In principle, the two-core fiber strain sensor can overcome this problem [22], however, it requires precise control of the critical fiber parameters in the manufacturing process. Therefore, the simple two-mode fiber strain gauge shows its special significance in the practicality. The simultaneous measurements of strain and temperature effects on a two-mode fiber is in progress.

Finally we discuss the resolution that can be achieved from this type of strain gauge. The elongation required to produce a 2π differential phase shift between the two spatial modes can be as small as the beatlength between them [16]. This can range from a few tens of micrometers up to a few hundred micrometers using typical fiber parameters. Consequently, highly elliptical core two-mode fiber strain sensors can potentially have resolutions covering more than an order of magnitude of range depending on the requirements for the particular application. The resolutions achievable from fibers having beat lengths on the short end of this spectrum is only one order of magnitude larger than that for two fiber-interferometers, and length changes as small as a fraction of an angstrom should be detectable.

V. CONCLUSION

This paper presents theoretical and/or experimental investigations of the propagation characteristics of both the fundamental and the second-order modes in fibers with highly elliptical cores under such perturbations as axial, radial, and hydrostatic strains, temperature change, twisting, squeezing, and bending.

The theoretical expressions developed here for the effects of perturbations apply to weakly guiding fibers and require experimentally measurable parameters; namely the polarization and modal birefringence, and their wavelength dispersion. Experimental results are supported by the theory except for the effects of intrinsic thermal stress and inhomogeneous acoustic properties across the cross section of the fiber. The difference in the response of the modal birefringence for the two eigenpolarization modes to an axial strain is utilized to build an interferometric strain gauge that provides a simultaneous measurement of the strain and the temperature. The modal fiber birefringence of the elliptical core fibers we tested had very low sensitivity to twist, squeeze, and bend, and a relatively low sensitivity to temperature change.

The unique mode characteristics of the two-mode fibers with highly elliptical core described in this paper could be useful for building new types of fiber devices such as sensors that are selectively sensitized for perturbations of interest and immune to others. Further studies with fibers with different design parameters are under way.

APPENDIX A

FIELD COMPONENTS OF ORTHOGONAL LP01 MODES AND ORTHOGONAL LP11 MODES IN A WEAKLY GUIDING CIRCULAR FIBER

\[
E_x^{(1)} = \begin{bmatrix}
J_0(\frac{ur}{a})/J_0(u), & 0, & -j \frac{w}{a} \frac{J_1(\frac{ur}{a})}{n k_0}
\end{bmatrix}
\cdot \cos \frac{\theta}{J_0(u)}, \quad \text{core}
\]
\[
K_0(\frac{wr}{a})/K_0(w), & 0, & -j \frac{w}{a} \frac{K_1(\frac{wr}{a})}{n k_0}
\end{bmatrix}
\cdot \cos \frac{\theta}{K_0(w)}, \quad \text{cladding}
\] (A1)

\[
E_y^{(1)} = \begin{bmatrix}
0, & J_0(\frac{ur}{a})/J_0(u), & -j \frac{w}{a} \frac{J_1(\frac{ur}{a})}{n k_0}
\end{bmatrix}
\cdot \sin \frac{\theta}{J_0(u)}, \quad \text{core}
\]
\[
0, & K_0(\frac{wr}{a})/K_0(w), & -j \frac{w}{a} \frac{K_1(\frac{wr}{a})}{n k_0}
\end{bmatrix}
\cdot \sin \frac{\theta}{K_0(w)}, \quad \text{cladding}
\] (A2)
\[
E_x^{(2)} = \left[ J_1 \left( \frac{\mu r}{a} \right) \cos \theta J_1 (u), -j \frac{\mu}{2n k_0} J_2 \left( \frac{\mu r}{a} \right) \cos \theta J_1 (u) \right] \text{, core}
\]
\[
\cdot \cos 2\theta - J_0 \left( \frac{\mu r}{a} \right) J_1 (u) \right], \text{ core}
\]
\[
K_1 \left( \frac{\nu r}{a} \right) \cos \theta \frac{K_1 (w)}{K_1 (w)} \left[ -j \frac{\nu}{2n k_0} K_2 \left( \frac{\nu r}{a} \right) \cos \theta \frac{K_1 (w)}{K_1 (w)} \right], \text{ cladding}
\]
\[
E_y^{(2)} = \left[ 0, J_1 \left( \frac{\mu r}{a} \right) \cos \theta J_1 (u), -j \frac{\mu}{2n k_0} J_2 \left( \frac{\mu r}{a} \right) \cos \theta J_1 (u) \right] \text{, core}
\]
\[
\cdot \sin 2\theta J_1 (u) \right], \text{ core}
\]
\[
0, K_1 \left( \frac{\nu r}{a} \right) \cos \theta \frac{K_1 (w)}{K_1 (w)} \left[ -j \frac{\nu}{2n k_0} K_2 \left( \frac{\nu r}{a} \right) \cos \theta \frac{K_1 (w)}{K_1 (w)} \right], \text{ cladding}
\]
where \( r, \theta \) are the cylindrical coordinates; \( a \) is the core radius; and \( n = n_1 = n_2 \). \( J \) is a Bessel function, and \( K \) is a modified Hankel function. The normalized parameters \( u \) and \( w \) have their conventional definitions.

REFERENCES

S.-Y. Huang received the B.S. degree in applied physics from Xian Jiao Tong University in Xian, China, in 1968. She received the M.S. and Ph.D. degrees from Shanghai Jiao Tong University in 1982 and 1985, respectively, both in electronic engineering. For her M.S. and Ph.D. studies she worked on fabrication and characterization of fiber-optic devices, as well as analyses and measurements of polarization characteristics in single-mode fibers. From 1986 to 1988, she was a Visiting Research Scholar at the Edward L. Ginzton Laboratory of Stanford University. During this period her research focused on fiber optic sensors. She is now Associate Professor of Shanghai Jiao Tong University. Her current interests are in optical-fiber communications and optical switching technology.

James N. Blake was born in Oakland, CA, in 1959. He received the B. S. E. E. degree from U. C. Berkeley in 1981, and the Ph. D. degree from Stanford University in 1988. From 1981 to 1984 he worked as a Microwave Engineer at Ford Aerospace in Palo Alto, CA. For his Ph.D. dissertation, he worked on two-mode optical fibers for sensing and signal processing applications. Currently, he is employed at Honeywell’s Systems and Research Center in Phoenix, AZ, working on all aspects of the fiber optic gyro.

Byoung Yoon Kim (S’83–M’85) was born in Seoul, Korea, in 1953. He received the B.S. degree from Seoul National University in 1977, and the M.S. degree from Korea Advanced Institute of Science in 1979, both in physics. He received the Ph.D. degree in applied physics from Stanford University in 1985. From 1979 to 1982, he was a Member of the Research Staff at Korea Institute of Science and Technology, Seoul, Korea, where he worked on fabrication and characterization of optical fibers, fiber-optic gyroscopes, polarization optical time domain reflectometry, and high precision laser interferometry. From 1985 to 1986 he was employed as a Research Associate at Edward L. Ginzton Laboratory, Stanford University, where he is currently an Acting Assistant Professor of Electrical Engineering. His current research interests are fiber-optic sensors and related components, optical signal processing, and guided wave modulators.