Passively mode-locked soliton lasers have been one of the dominant means to generate ultrashort pulses in fiber laser systems, together with stretched-pulse lasers and the recently demonstrated similariton lasers [1–3]. The fundamental soliton condition [4], which requires a balance between cavity anomalous dispersion and Kerr nonlinearity, limits the pulse energy and the pulse duration to certain levels. This peak-power-limiting effect, described by the soliton area theorem [1], thus naturally causes multiple pulsing of quantized soliton per round trip as the pump power increases [5]. Kutz et al. showed that forward pulse drift due to time-dependent gain depletion can be balanced by the amount of recovered population inversion between pulses, resulting in equidistant passive harmonic mode-locking (H-ML) [6]. This is a very simple way to multiply the repetition rate up to the GHz range without needing to reduce the fiber cavity length to impractical levels or requiring any active GHz modulators. This kind of optical source would be useful for applications in wavelength division multiplexing (WDM) communication via post-spectral broadening [7] or astronomical frequency combs via post-amplification [8] provided that the repetition rate can be sufficiently increased and the stability is guaranteed. So far, nonlinear polarization rotation (NPR) has been adopted as the primary means of achieving passive H-ML in Yb [9], double-clad Yb [10], Er:Yb [11], and double-clad Er:Yb fibers [12,13]. Most of these, however, required pump powers of several to tens of watts to generate repetition rates of several GHz, showing a low pumping efficiency of <2.5 MHz/mW and a super-mode suppression ratio (SMSR) that was not higher than 30 dB, except in Ref. 9. Moreover, the optical spectra showed large continuous-wave (CW) peaks by which up to several tens of percent of the total power was wasted. Colliding-pulse configuration with an Yb fiber and a semiconductor saturable absorber mirror (SESAM) showed better performance with an SMSR of >60 dB, but limited to the 14th harmonics at a repetition rate of 605 MHz [14]. Recently, passive H-ML using an Er fiber and a carbon nanotube (CNT) saturable absorber (SA) was also demonstrated up to 23rd order, 328.44 MHz with a SMSR of 30 dB [15]. However, the ferrule-type SA used is inherently subject to optical damage, which limits the pump power level to 60 mW. As an alternative, we recently demonstrated low-noise GHz (943.16 MHz at 34th harmonics) passive H-ML with 195 mW of pump power using a CNT SA of the evanescent-wave interaction type [16]. For efficient utilization of the pump power to further increase the repetition rate, it is important to systematically investigate the key parameters affecting the maximum harmonic order of passive H-ML.

In this Letter, we demonstrate multi-GHz passive H-ML with moderate pump power by engineering the soliton pulse energy in an Er-doped fiber (EDF) laser incorporating a CNT SA interacting via evanescent waves. First, we experimentally study the influence of cavity dispersion and spectral bandwidth on the soliton pulse energy and the increase in the harmonic order of passive H-ML based on the soliton area theorem. As a result, we obtain a very efficient repetition rate slope of >13 MHz/mW for a given pump power. Integrating all the characterization trends with a high-gain EDF laser, we can eventually acquire a repetition rate of ~5 GHz and a SMSR of 40 dB with only ~400 mW of pump power in a single-clad fiber laser.

In the soliton area theorem [1], the averaged cavity soliton can be expressed as

$$A_0 \tau = \sqrt{\frac{2[D]}{\delta}} \quad \text{with} \quad D \equiv \frac{1}{2} \beta_{2,\text{ave}} L, \quad \delta = \gamma_{\text{ave}} L,$$

where $A_0$ is the peak amplitude, $\tau$ is the pulse duration at $1/e$, $\beta_{2,\text{ave}}$ is the average cavity dispersion, $L$ is the cavity length, and $\gamma_{\text{ave}}$ is the average nonlinear parameter. By squaring both sides, Eq. (1) can be expressed as the well-known fundamental soliton condition as follows:

$$P \tau^2 = P \left( \frac{T_{\text{FWHM}}}{1.7627} \right)^2 = \frac{\alpha}{1.7627^2} E T_{\text{FWHM}} = \left[ \frac{\beta_{2,\text{ave}}}{\gamma_{\text{ave}}} \right],$$

where $P$ is the peak power, $\alpha$ is the peak-power conversion factor (0.88 for sech²), and $T_{\text{FWHM}}$ is the pulse duration at full-width at half maximum. Therefore, by decreasing the average cavity dispersion of the laser, the average soliton energy can be lowered for transform-limited pulses with fixed optical spectrum bandwidth. Although this concept was previously applied in passive H-ML to obtain higher repetition rates [5,10], there has
been no systematic investigation to optimize the laser cavity condition.

The experimental setup is shown in Fig. 1(a), similar to that of [16]. An EDF (small signal gain of 17 dB/m at 1530 nm) with a length of 2.9 m and a CNT SA based on evanescent field interaction are used for laser operation. The optical spectrum, radio-frequency (RF) spectrum, time domain waveform, and output power were simultaneously monitored. The pulse duration was measured with an auto-correlator. The harmonic order increases stepwise as the pump power increases, where the state of polarization in the laser cavity is slightly adjusted at each harmonic order. Once passive H-ML is established, it stably operates for a few hours in our experiment. Figures 1(b) and (c) show the results of passive H-ML for different cavity dispersions. The RF spectra of the laser outputs are compared for the cases of average cavity dispersion of −8.0 ps²/km and −3.9 ps²/km. With the higher dispersion value of Fig. 1(b), the measured soliton energy was as high as 21.2–24.7 pJ, and 254 mW of pump power generated 40th order, 787.6 MHz from the 19.69 MHz fundamental mode spacing. On the other hand, the lower dispersion value of Fig. 1(c) resulted in 5.9–6.9 pJ of pulse energy, where the repetition rate was greatly enhanced to 77th order, 1.925 GHz H-ML from the 25 MHz fundamental mode with 161 mW of pump power. This result is consistent with previous predictions. Figure 2 shows the optical spectrum and auto-correlation trace for the case of the 70th harmonic order at 1.75 GHz (shown in the second row of Fig. 1(c)), which shows a 3 dB bandwidth of 7.9 nm and a pulse width of 348.9 fs with a sech² shape. The time-bandwidth product (TBP) is 0.338. In all cases of H-ML with various cavity dispersions, optical spectra were similarly maintained between the 3 dB bandwidths of 7.4–8.9 nm with the clear spectral characteristics of soliton pulses, and the TBPs were nearly transform-limited between 0.319 and 0.338.

Figure 3 summarizes the results of passive H-ML as a function of pump power and average cavity dispersion, respectively. By decreasing the average cavity dispersion from −8.0 ps²/km (black) to −6.6 (red), −5.1 (blue), and −3.9 ps²/km (green), we can clearly see that average soliton energy is decreased, resulting in a steeper slope of the passive H-ML from 3.1 MHz/mW to 6.2, 9.2, and 13.1 MHz/mW, respectively. The corresponding fundamental mode spacing was 19.69 MHz, 21.50 MHz, 23.55 MHz and 25.00 MHz. A further decrease of the cavity dispersion to −3.3 ps²/km showed the feature of the stretched-pulse lasers, in which the optical spectrum and pulse energy gradually increased as the pump increased. Throughout this verification, we could eventually acquire a 77th order, 1.925 GHz repetition rate with only 161 mW of pump power. The slope efficiency of 13.1 MHz/mW is remarkably higher than previous research efforts [9–16], and this characterization suggests a method for efficient utilization of pump power in higher-order passive H-ML.

The soliton pulse energy could also be lowered by the polarization control of intra-cavity due to the polarization dependence of our CNT SA [16,17]. By adjusting the polarization controller carefully, discrete sets of stable H-ML could be acquired which had different spectral widths (8–10.3 nm, 5.2–7.2 nm, and 3.3–5.2 nm) and pulse energies (30.2–35.2 pJ, 11.8–14.3 pJ and 6.1–9.2 pJ), respectively, as shown in Fig. 4. Cavity nonlinearity and average dispersion (−8.0 ps²/km) are fixed in this case. We speculate that this behavior originates from different modulation depths and saturation fluences of the CNT SA according to the polarization state of the light incident on the SA, which yield different pulse-shaping ability resulting in several versions of the optical spectrum. Then, from Eq. (2), a narrower optical spectrum (broadened pulse duration) generates a smaller pulse energy if the

![Fig. 1. (Color online) (a) Schematic setup. RF spectrum of passive H-ML as the pump power increases when the average (Ave.) cavity dispersion is (b) −8.0 ps²/km and (c) −3.9 ps²/km.](image1)

![Fig. 2. (Color online) (a) Optical spectrum and (b) auto-correlation trace when the average cavity dispersion is −3.9 ps²/km and pump power is 148 mW (70th, 1.75 GHz H-ML).](image2)

![Fig. 3. (Color online) Passive H-ML slopes as a function of average cavity dispersion. The annotations indicate the average cavity dispersion, pulse energy, and 3 dB optical spectrum. The lines are linear-fittings.](image3)
TBP is not severely changed. As a result, the fiber laser with the narrower spectrum of $3.3-5.2$ nm increased its harmonic order up to the 91st, 1.792 GHz with 199 mW pump power, which is about 3 times more efficient than the case of the 8–10.3 nm as shown in Fig. 4. The SMSR was maintained higher than 40 dB during all experiments in this work. Additional noise characterization such as relative intensity noise and timing jitter is in progress. It should be noted that our experimental results, such as spectral properties, SMSR, and self-arrangement between harmonic pulses, critically depend on the performance of the CNT SA. The measurement of the modulation depth and saturation fluence of the SA is still under investigation because it is not easy to characterize the in-line SA having polarization-dependent loss, due to the effect of NPR in a fiber during the measurement. The use of an in-line SA composed of polarization-maintaining fiber can be considered for precise characterization of the SA and robust operation of fiber lasers as a future work.

Finally, we built a high gain (small-signal gain of 96 dB/m at 1530 nm) EDF soliton laser with average dispersion of $-10.5$ ps$^2$/km. A compromise was made in the tailoring of the cavity dispersion to maintain a soliton regime, but narrowing the 3-dB optical spectrum at 2.01 nm and the larger nonlinear parameter of the EDF helped reduce the pulse energy to 4 pJ. With an efficiency of 13.8 MHz/mW, 128th of 38.4 MHz, 4.915 GHz repetition rate was achieved with 395 mW pump power, which is about 3 times more efficient than the case of the 8–10.3 nm as shown in Fig. 5. The inset of Fig. 5 shows the SMSR of $\sim 40$ dB.

In conclusion, we have demonstrated a systematic approach to increasing the repetition rate of passive H-ML. The influence of the cavity dispersion and spectral bandwidth on the increase of the harmonic order has been studied. By reducing the pulse energy, we can acquire a steep slope efficiency of $\sim 13$ MHz/mW which is far higher than achieved in previous works. Integrating all the characterization trends with a high-gain EDF laser, we acquire a $\sim 5$ GHz repetition rate and a $\sim 40$ dB SMSR with $\sim 400$ mW of pump power. This is an exceptional result which shows the possibility of a repetition rate $>10$ GHz with $<1$ W of pump power in a single-clad fiber laser.

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